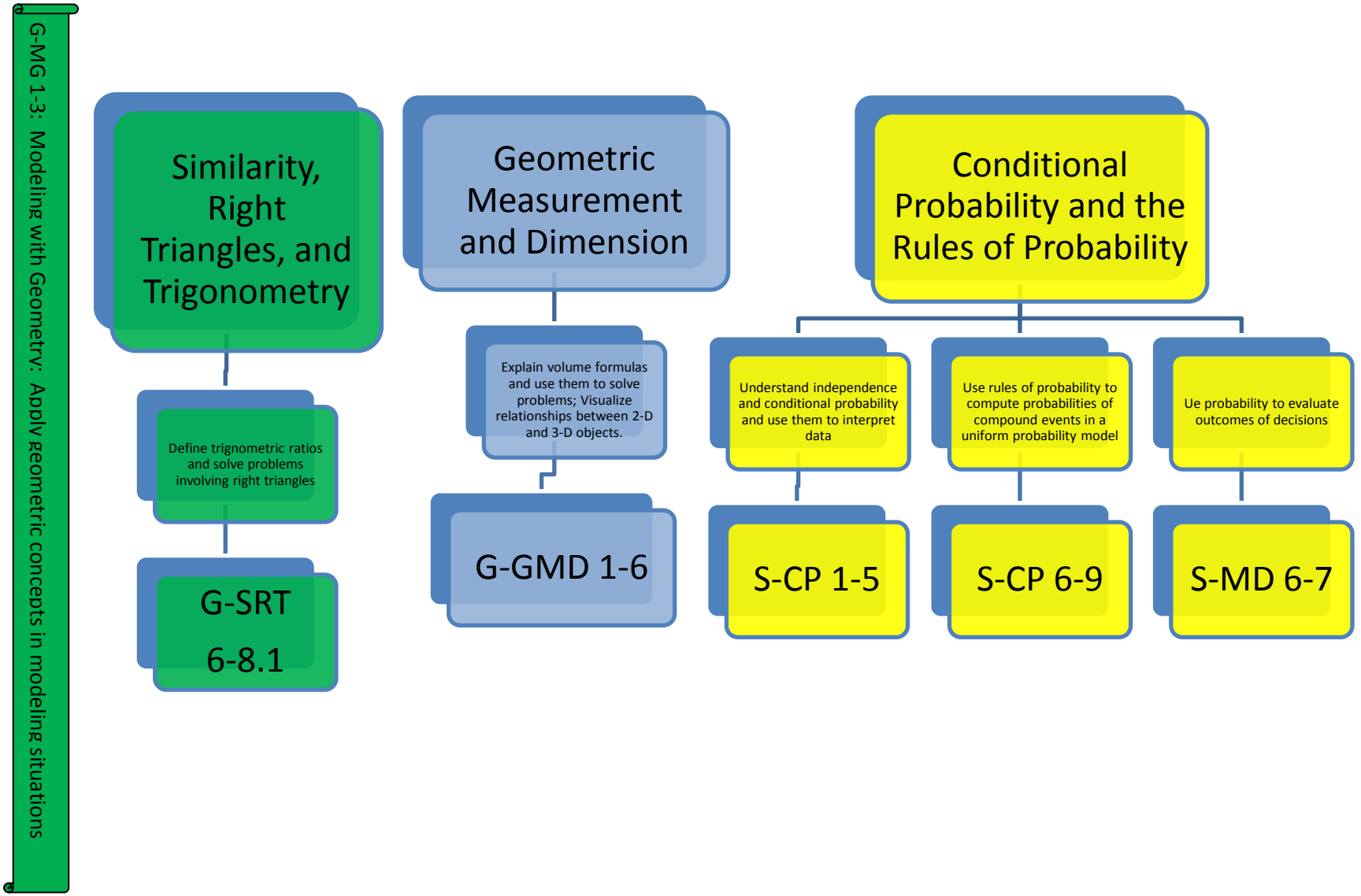
# High School Geometry

## Unit 4




Key: Major Clusters; Supporting Clusters; Additional Clusters

**High School Geometry – UNIT 4**
**Trigonometry; Measurement and Dimensions; Statistics and Probability**

**Critical Area**: Students explore probability concepts and use probability in real-world situations. They continue their development of statistics and probability, students investigate probability concepts in precise terms, including the independence of events and conditional probability. They explore right triangle trigonometry, and circles and parabolas. Throughout the course, Mathematical Practice 3, "Construct viable arguments and critique the reasoning of others," plays a predominant role. Students advance their knowledge of right triangle trigonometry by applying trigonometric ratios in non-right triangles.

| CLUSTERS | COMMON CORE STATE STANDARDS |
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| **Define trigonometric ratios and solve problems involving right triangles.** | **Geometry - Similarity, Right Triangles, and Trigonometry**<br>**G.SRT.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.<br><br>**G.SRT.7** Explain and use the relationship between the sine and cosine of complementary angles.<br><br>**G.SRT.8** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.<br><br>**G.SRT.8.1** Derive and use the trigonometric ratios for special right triangles (30°,60°,90°and 45°,45°,90°). CA |
| **Explain volume formulas and use them to solve problems**<br><br>**Visualize relationships between two-dimensional and three-dimensional objects**. | **Geometric Measurement and Dimension**<br>**G.GMD.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*<br><br>**G.GMD.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.<br><br>**G.GMD.4** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.<br><br>**G.GMD.5** Know that the effect of a scale factor k greater than zero on length, area, and volume is to multiply each by k, $k^2$, and $k^3$, respectively; determine length, area and volume measures using scale factors. CA<br><br>**G.GMD.6** Verify experimentally that in a triangle, angles opposite longer sides are larger, sides opposite larger angles are longer, and the sum of any two side lengths is greater than the remaining side length; |

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| | apply these relationships to solve real-world and mathematical problems. CA |
| **Understand independence and conditional probability and use them to interpret data** (Link to data from simulations or experiments.) | **Statistics and Probability - Conditional Probability and the Rules of Probability**<br><br>**S.CP.1** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").<br><br>**S.CP.2** Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.<br><br>**S.CP.3** Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.<br><br>**S.CP.4** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*<br><br>**S.CP.5** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. |
| **Use the rules of probability to compute probabilities of compound events in a uniform probability model** | **Statistics and Probability - Conditional Probability and the Rules of Probability**<br><br>**S.CP.6** Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.<br><br>**S.CP.7** Apply the Addition Rule, *P*(*A* or *B*) = *P*(*A*) + *P*(*B*) – *P*(*A* and *B*), and interpret the answer in terms of the model.<br><br>**S.CP.8** (+) Apply the general Multiplication Rule in a uniform probability model, *P*(*A* and *B*) = *P*(*A*)*P*(*B*\|*A*) = *P*(*B*)*P*(*A*\|*B*), and interpret the answer in terms of the model. |

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| | **S.CP.9** (+) Use permutations and combinations to compute probabilities of compound events and solve problems. |
| MATHEMATICAL PRACTICES | |
| **1. Make sense of problems and persevere in solving them.**<br>**2. Reason abstractly and quantitatively.**<br>**3. Construct viable arguments and critique the reasoning of others.**<br>**4. Model with mathematics.**<br>5. Use appropriate tools strategically.<br>6. Attend to precision.<br>7. Look for and make use of structure.<br>8. Look for and express regularity in repeated reasoning. | Emphasize Mathematical Practices 1, 2, 3, and 4 in this unit. |
| **LEARNING PROGRESSIONS** | |
| Draft High School Progression on Statistics and Probability<br>http://ime.math.arizona.edu/progressions/ | |

**★Indicates a modeling standard linking mathematics to everyday life, work, and decision-making.**

**(+) Indicates additional mathematics to prepare students for advanced courses.**

| ENDURING UNDERSTANDINGS | ESSENTIAL QUESTIONS | KEY VOCABULARY |
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| • Understand trigonometric ratios as the relationships between sides and angles in right triangles.<br>• Understand the concept of complementary angles through sine and cosine.<br>• Trigonometric ratios can be derived for special right triangles (30-60-90 and 45-45-90).<br>• Real world problems can be solved using right triangles, trigonometric ratios and the Pythagorean theorem.<br>• The formulas for circumference of a circle, area of a circle; volume of a cylinder, pyramid and cone can be derived using informal reasoning and solve real-world problems involving the volume for cylinders, pyramids, cones and spheres.<br>• The 2-dimensional shapes formed from the cross- | • Based on similarity, how can you connect the concept of side ratios as angle properties to define the three trigonometric ratios?<br>• Using the concept of complementary angles, how are sine and cosine related?<br>• What generalizations can be made about how you can use an equilateral triangle and the Pythagorean Theorem to make generalizations about the 3 trigonometric ratios for special right triangles?<br>• How do you develop the circumference of a circle, area of a circle, volume of a cylinder, pyramid and cone using informal arguments (i.e. | • Addition Rule<br>• Cavalieri's Principle<br>• Circumference<br>• Combination<br>• Complementary<br>• Compound event<br>• Conditional probability<br>• Cone<br>• Cosine<br>• Cross-section<br>• Cylinder<br>• Dependent/independent variable<br>• Derive<br>• Independent probability<br>• Informal Argument |

| ENDURING UNDERSTANDINGS | ESSENTIAL QUESTIONS | KEY VOCABULARY |
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| sections of a 3-dimensional object and the 3-dimensional object formed by rotating a 2-dimensional object is defined.<br>• A scale factor ($k > 0$) can affects the length, area and volume of an object.<br>• How angle measures correspond to side lengths in a triangle. (i.e. smallest angle measures are opposite shortest side lengths)is demonstrated.<br>• Triangle Inequality Theorem is verified using measurement.<br>• Conditional probability of A given B as the fraction of B's outcomes that also belong to A, is interpreted and modeled.<br>• Permutations and combinations probabilities of compound events is computed and used to solve problems.<br>• The addition and general multiplication rule can be applied and interpret probability models | paper folding/cutting)?<br>• What generalizations can be made about the cross-sections of 3-dimensional objects and rotations formed from 2-dimensional objects?<br>• How can you use scale factor to determine the length, area, and volume of similar objects?<br>• What generalizations can be made about the relationship between side lengths and angle measures and also the relationship between side lengths?<br>• How can you use triangle inequality theorem and relationship between side lengths and angles measures to solve real-world problems?<br>• How can you explain the concepts of conditional probability and independence in everyday language and everyday situations?<br>• How is permutations and combinations probabilities of compound events used in problem solving?<br>• What interpretation can be made of probabilities' addition and general multiplication rule? | • Multiplication Rule<br>• Outcomes<br>• Permutation<br>• Pyramid<br>• Pythagorean Theorem<br>• Rotation<br>• Scale Factor<br>• Similarity<br>• Sine<br>• Sphere<br>• Tangent<br>• Trigonometric Ratios<br>• Uniform probability<br>• Volume |

| RESOURCES | INSTRUCTIONAL STRATEGIES | ASSESSMENT |
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| **LAUSD Adopted Textbooks and Programs**<br>• Big Ideas Learning - Houghton Mifflin Harcourt, 2015: Big Ideas Geometry<br>• College Preparatory Mathematics, 2013: Core Connections, Geometry<br>• The College Board, 2014:Springboard Geometry | Create an informative poster with a T-table explaining when to use permutation or combination formula.<br><br>Teach students the acronym SOH-CAH-TOA so that they can easily remember the trigonometric ratios. | LAUSD ASSESSMENT<br>The district will be using the SMARTER Balanced Interim Assessments. Teachers would use the Interim Assessment Blocks (IAB) to monitor the progress of students. Each IAB can be given twice to show growth over time. |

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| **Illustrative Mathematics**<br>Defining Trigonometric Ratios: G.SRT.6<br>http://www.illustrativemathematics.org/illustrations/1635<br>Sine and Cosine of Complementary Angles: G.SRT.7<br>http://www.illustrativemathematics.org/illustrations/1443<br>Shortest line segment from a point *P* to a line *L:* G.SRT.8<br>http://www.illustrativemathematics.org/illustrations/962<br>Doctor's Appointment: G.GMD.3<br>http://www.illustrativemathematics.org/illustrations/527<br>Centerpiece: G.GMD.3<br>http://www.illustrativemathematics.org/illustrations/514<br>Area of a circle: G.GMD.1<br>http://www.illustrativemathematics.org/illustrations/1567<br>Global Positioning System: G.GMD.4, A.CED.2<br>Ihttp://www.illustrativemathematics.org/illustrations/1215<br>Rain and Lightning:S.CP.2,3,5, and 7<br>http://www.illustrativemathematics.org/illustrations/1112<br>Lucky Envelopes: S.CP.3<br>http://www.illustrativemathematics.org/illustrations/944<br>Random Walk: S.CP.9<br>http://www.illustrativemathematics.org/illustrations/689<br>**Illuminations** | Retrieve actual 3-D items (orange, rectangular cake, cheese block etc.) and demonstrate how a cross-section is like cutting these items and how the 2-D shape can be seen after the cut.<br><br>Post up a problem on probability of compound events on the board for students to solve independently. Using the four walls of the classroom, post different possible solutions that students might arrive at. Students will walk to the wall that has their answer. Each group of students will have to defend their answer by explaining how they got their answer and justify why they are correct. | STATE ASSESSMENT<br><br>California will be administering the SMARTER Balance Assessment as the end of course for grades 3-8 and 11. There is no assessment for Algebra 1. The 11th grade assessment will include ítems from Algebra 1, Geometry, and Algebra 2 standards. For examples, visit the SMARTER Balance Assessment at:<br>http://www.smarterbalanced.org/<br>Sample Smarter Balanced Items:<br>http://sampleitems.smarterbalanced.org/itempreview/sbac/index.htm |

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| Trigonometry for Solving Problems http://illuminations.nctm.org/LessonDetail.aspx?id=L383 | | |

LANGUAGE GOALS for **low achieving, high achieving, students with disabilities and English Language Learners**

- Students will identify words in probability word-problems that will help them solve them using a causative structure like: *The following words (____ and ___) help me solve the problem* or *The words ____ and ____ help me solve the problem.*
- Students will record step-by-step directions for finding the volume of solid figures using transition words like "first," "second," "next" and "finally."
- Students will describe their understanding of a two-way frequency table, using the words *relative*, *percent*, *column/row,* and *dependent/independent events*.
- Students will describe the shapes of two-dimensional cross-sections of three-dimensional objects, and of three-dimensional objects generated by rotations of two-dimensional objects.
- Students will explain and use the relationship between the sine and cosine of complementary angles.
- Students will write a few sentences describing a specific way to use permutation or combination to compute probability of compound events to solve a problem, linking their opinion and reasons using specific words and phrases (such as consequently, and specifically).

PERFORMANCE TASKS

**Mathematics Assessment Project**

Funsize cans: G-GMD.3 **http://map.mathshell.org/materials/download.php?fileid=756**

Glasses: G-GMD.3 http://map.mathshell.org/materials/tasks.php?taskid=259#task259

Bestsize Cans: G-GMD.3 http://map.mathshell.org/materials/tasks.php?taskid=284#task284

Propane Tanks: G-GMD.3 http://map.mathshell.org/materials/tasks.php?taskid=288#task288

Hopewell Geometry: G-SRT.5 , 6, and 8 http://map.mathshell.org/materials/tasks.php?taskid=127#task127

**Illustrative Mathematics**

Coins in a circular pattern: G.SRT.8, G.MG http://www.illustrativemathematics.org/illustrations/720

Seven Circles III: G.SRT.8, G.SRT.D, G.MG.1 http://www.illustrativemathematics.org/illustrations/710

Ask a pilot: G.SRT.8 http://www.illustrativemathematics.org/illustrations/1638

Use Cavalieri's Principle to Compare Aquarium: G.GMD.2, G.MG.1. http://www.illustrativemathematics.org/illustrations/530

Tennis Balls in a Can: G.GMD.4, G.MG.1 http://www.illustrativemathematics.org/illustrations/512
Global Positioning System II: G.GMD.4, G.MG.1 http://www.illustrativemathematics.org/illustrations/1202
The Titanic 1: S.CP.1,4, and 6 http://www.illustrativemathematics.org/illustrations/949
The Titanic II: S.CP.2-6 http://www.illustrativemathematics.org/illustrations/950
Return to Fred's Fun Factory (with 50 cents): S.CP.1,2, and 9 http://www.illustrativemathematics.org/illustrations/1198
Law of Sines and Law of Cosines: http://illuminations.nctm.org/LessonDetail.aspx?ID=L703

| DIFFERENTIATION📖 | | |
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| **UDL/FRONT LOADING** | **ACCELERATION** | **INTERVENTION** |
| Prerequisites:<br>• Review and have students provide examples of proportion and ratios. They can construct a 3-D solid and copy within specific proportions.<br>• Have students should review similar triangles.<br>• Vocabulary should be reviewed<br>• Engage students in a discussion about planes versus space (2-D versus 3-D) as well as area formulas and how to use them to find the volume formulas.<br>• Use T-chart or other graphic organizer to compare Independent Events and Dependent Events.<br><br>Use Frayer model to provide the definition of probability. | • Design an activity where students would collect data and then use probability model to interpret the data. **For example**, students can collect data to answer the following real-life question: There is little doubt that caffeine stimulates bodily activity, but how much does it take to produce a significant effect? This is a question that involves measuring the effect of two or more treatments and deciding if the different interventions have differing effects. To obtain a partial answer to the question on caffeine, it was decided to compare a treatment consisting of 200 mg of caffeine with a control of no caffeine in an experiment involving a finger tapping exercise.<br>Twenty male students were randomly assigned to one of two treatment groups of 10 students each, one group receiving 200-milligram of caffeine and the other group no caffeine. Two hours later the students were given a finger tapping exercise. The response is<br>the number of taps per minute, as shown in the table. | • Hands-on 3 D solids that allow student to have the visual to understand different parts and vocabulary of volumes.<br>• Also the hands on will allow volume comparison.<br>• Interactive online websites describing the changes in ratio with changing dimensions.<br>• Scaffolding<br>• Vocabulary wall<br>• To increase active participation, students should be expected to work collaboratively to help language learners with lowering anxiety, promote authentic conversation, opportunities for asking questions, and support peers and teachers.<br><br>Alex, Mel, and Chelsea play a game that has 6 rounds. In each round there is a single winner, and the outcomes of the rounds are independent. For each round the probability that Alex wins is 12, and Mel is twice as likely to win as Chelsea. What is the probability that Alex wins three rounds, Mel wins two rounds, and Chelsea wins one round?<br>http://www.illustrativemathematics.org/illustrations/1035 |

| DIFFERENTIATION📖 | | |
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| **UDL/FRONT LOADING** | **ACCELERATION** | **INTERVENTION** |
| | **Finger taps per minute in a caffeine experiment**<br>0 mg caffeine / 200 mg caffeine<br>242 / 246<br>245 / 248<br>244 / 250<br>248 / 252<br>247 / 248<br>248 / 250<br>242 / 246<br>244 / 248<br>246 / 245<br>242 / 250<br>Mean 244.8 / 248.3<br>Source: Draper and Smith, *Applied Regression Analysis*. John Wiley and Sons, 1981<br><br>• Students use trigonometric functions to find dimensions or distances of objects in real life. **For instance**: Around 1:30 p.m., people heard that the space shuttle will fly around Los Angeles area. People were outside waiting. Finally, the space shuttle is observed. At one point, it appeared as if the shuttle was really low. The observer's distance is about 100 feet away from it (diagonal distance) with an angle of elevation of $30^{o}$. How high is the shuttle from the ground?<br><br>• Use the following activity which requires students to identify whether or not a game is fair: http://insidemathematics.org/problems-of-the-month/pom-gameshow.pdf | |

[1] **Major Clusters – area of intensive focus where students need fluent understanding and application of the core concepts.**

[2] **Supporting/Additional Clusters – designed to support and strengthen areas of major emphasis/expose students to other subjects.**

**References:**


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